EFFECTIVE THERMAL DIFFUSIVITY OF A VIBRATIONALLY FLUIDIZED BED

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UDC 536.2

Studies have been made on the effective thermal diffusivity in an extended vibrationally fluidized bed with or without a set of vertical tubes.

Internal heat transfer occurs in an extended vibrationally fluidized bed, e.g., in a horizontal apparatus of boat type, and may be characterized from the effective thermal diffusivity [1]. If the bed contains bundles of vertical tubes to remove or supply heat [2], the large-scale circulation is conserved only in the transverse direction, which can tend to equalize the temperatures. Along the bed, the transport is mainly via small-scale fluctuations, so one can take the thermal conductivity as one-dimensional. A similar situation occurs without the inserts with developed vibrational fluidization.

A nonstationary method has been used to examine the effective thermal diffusivity, which is based on the thermal conduction solution for an unbounded rod with a heat-transfer side surface and instantaneous heat sources distributed over a certain past [3]. The experiments were performed in a trough-type apparatus with dimensions 600 × 110 × 150 mm, made of thermally insulating material. The apparatus was divided lengthwise by a mobile baffle into two chambers: a hot one having length $\underset{\sim}{R}$ = 100 mm and a cold one 500 mm long. The hot chamber was loaded with material heated to $T^* = 90\text{-}120°C$, while the temperature of the bed in the cold chamber was the environmental temperature T_0.

When the barrier was removed, the temperatures were recorded in three sections along the cold chamber at distances x = 130, 170, and 210 mm. Figure 1 shows typical results. The effective thermal diffusivity was determined from the time required to reach the maximum temperature. These values of a for the free layer or with the inserts were $1.3\text{-}13 \times 10^{-5}$ m²/sec and were larger by one or two orders of magnitude than the result for an immobile bed (a_0 - 0.05×10^{-5} m²/sec) because of the random motion.

In general, a is dependent on various factors (A, ω_0, d, H_0, etc.), of which the most important is the vibrational frequency ω_0; $a(\omega_0)$ has a pronounced maximum. For example, with a bed 60 mm deep, the peak occurred at $\omega_{0m} = 220$ sec^{-1}.

The effective thermal diffusivity is represented for theoretical purposes as

$$a = a_0 + D_\omega(\omega_0) \tag{1}$$

(for $\omega_0 \leqslant \sqrt{g/A} \quad D_\omega(\omega_0) \to 0, a \to a_0$).

D_ω is the temperature-diffusion coefficient and can be considered as the analog of the turbulent thermal diffusivity as determined by the rate of the small-scale random movements. A particle moving on a random path transports the temperature as a passive scalar trace component while exchanging heat with the environment. Then D_ω can be determined from the theory of turbulent transport [4]. The autocorrelation function for the random particle velocities $R_V(t)$ gives for sufficiently long times [4] that

$$D_\omega = \int_0^\infty R_V(t) f(\alpha, t)\, dt. \tag{2}$$

The function $f(\alpha, t)$ incorporates the heat transfer between the particles and the environment. If the behavior is coherent, (2) can be used for a group of particles, as has been done [5] for the thermal diffusivity in a fluidized bed.

Kirov Urals Polytechnical Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 4, pp. 610-618, April 1990. Original article submitted February 21, 1989.

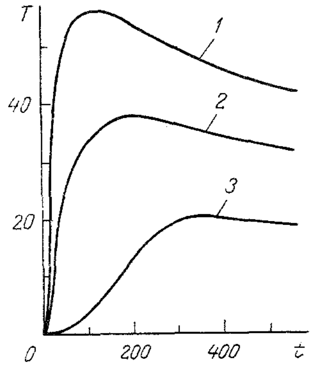

Fig. 1. Bed temperature T (°C) as a function of time t (sec) in a vibrationally fluidized bed of electrocorundum having d = 0.16 mm and with a corridor set of vertical tubes 10 mm in diameter placed with pitch $s_1 = s_2 = 20$ mm and used at f = 35 Hz with A = 0.8 mm for the following sections: 1) x = 130 mm; 2) 170; 3) 210.

The heat transfer can be described in the relaxation approximation, where f(α, t) can be based on Burger's results as given in [4]:

$$\frac{dT_p}{dt} = \alpha\Delta T, \quad f(\alpha,\ t) = \exp^{-\alpha t}. \tag{3}$$

A cosine transformation $R_V(t) = F_V(\omega)$ is used with the substitution of (3) for f(α, t) into (2) to give

$$D_\omega = 2\int_0^\infty\int_0^\infty F_V(\omega)\exp^{-\alpha t}\cos(\omega t)\,d\omega dt.$$

Integration with respect to t gives

$$D_\omega = 2\int_0^\infty F_V(\omega)\,\frac{\alpha}{\omega^2+\alpha^2}\,d\omega. \tag{4}$$

To determine $F_V(\omega)$, we use the equation of motion in projection on the horizontal x axis:

$$m_p\ddot{x} = P_1 + P_2 + P_3. \tag{5}$$

The forces on the right in (5) are: P_1 the dissipative resistance, which is taken as a linear function of the velocity, $P_1 = -2\lambda m_p\dot{x}$, where the resistance coefficient in general is a function of frequency, $\lambda = \lambda(\omega_0)$; P_2 the force due to the gas elasticity, as the pressure oscillates with frequency Ω [6], and in accordance with [7] $P_2 = -\Omega^2 m_p x$; and $P_3 = m_p f_p$ the force representing the random action of the environment on the particle.

The explicit forms of P_1, P_2, P_3 give (5) in the coordinate and Fourier representations as

$$\ddot{x} + 2\lambda\dot{x} + \Omega^2 x = f_p, \tag{6}$$

$$[(i\omega)^2 + 2\lambda(i\omega) + \Omega^2]\,x_\omega = f_{p\omega}. \tag{7}$$

An equation analogous to (6) describes a rheological model for a vibrationally fluidized bed if f_p is a harmonic force [6]. One uses (6) and the complex conjugate with ensemble averaging by standard methods [8, 9] to get the spectral densities for the random displacements

$$F_x(\omega) = x_\omega^2 = \frac{F_p(\omega)}{(\omega^2 - \Omega^2)^2 + 4\lambda^2(\omega)\,\omega^2} \tag{8}$$

and velocities

$$F_V(\omega) = \omega^2 F_x(\omega). \tag{9}$$

Here $F_p(\omega) = (f_{p\omega})^2$ is the spectral density of the random force, which is constant for white noise. Then (8) and (9) describe the frequency spectra for a Brownian oscillator [8].

Assumptions [5, 10] on the random forces in a fluidized bed lead to results confirmed by experiment. As we lack information on such forces on a particle in a vibrational bed, we follow [5] and assume that these are random pulses $f_p(t) = \sum_k b_k F(t - t_k)$ and if elastic interactions predominate, $f_p(t)$ will be a narrow-band random process whose spectrum can [11] be put as

$$F_p(\omega) = \frac{1}{2} A \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}. \tag{10}$$

We substitute (9) and (10) into (4) to get

$$D_\omega = A\alpha\Gamma \int_0^\infty \frac{\omega^2 d\omega}{[(\omega^2 - \Omega^2)^2 + 4\lambda^2\omega^2]\,[(\omega - \omega_0)^2 + \Gamma^2]\,(\omega^2 + \alpha^2)}. \tag{11}$$

If there is little attenuation in the force pulses acting on a particle, $\Gamma \ll \omega_0$, which enables one to use the asymptote [12]

$$\frac{1}{\pi} \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2} \approx \delta(\omega - \omega_0). \tag{12}$$

Then (11) and (12) give

$$D_\omega = \frac{\pi A \alpha \omega_0^2}{[(\omega_0^2 - \Omega^2) + 4\lambda_\omega \omega_0^2]\,(\omega_0^2 + \alpha^2)}, \tag{13}$$

in which $\lambda_\omega = \lambda(\omega_0)$.

If the width $D_\omega(\omega_0)$ is small, as is observed (Fig. 2), the peak in D_ω lies near Ω [12], so we put $\omega_{0m} \approx \Omega$, and then the maximal thermal diffusivity $D_\omega(\omega_{0m}) = D_{\omega m}$, with (13) gives

$$\pi A \alpha = 4\lambda_m^2 (\omega_{0m}^2 + \alpha^2) D_{\omega m},$$

in which $\lambda_m = \lambda_m(\omega_{0m})$. We use this with $\eta = \omega_0/\omega_{0m}$, $\gamma = \alpha/\omega_{0m}$, $\varkappa_\omega = \omega_{0m}/2\lambda_\omega$, $\varkappa_m = \omega_{0m}/2\lambda_m$, to transform (13) to

$$\frac{D_\omega}{D_{\omega m}} = \frac{(1 + \gamma)^2 \eta^2}{[\varkappa_m^2 (\eta^2 - 1)^2 + (\varkappa_m/\varkappa_\omega)^2 \eta^2]\,(\gamma^2 - \eta^2)}. \tag{14}$$

We estimate γ^2 in (14). For solid particles (corundum, $d = 7 \cdot 10^{-5}$ m, $\rho_p = 3.9 \cdot 10^3$ kg/m³, $c_p = 780$ J/kg·K) exchanging heat with a gas (air, $\lambda_{qf} = 2.6 \cdot 10^{-2}$ W/m·K), the time-constant in (3) can [13] be put as $\alpha = 12\lambda_{qf}/(\rho c_p d)^2$ and is approximately 20 sec⁻¹. That is larger by an order of magnitude than the α calculated from the heat-transfer coefficient in a blown vibrational layer [14]. As $D_{\omega m}$ occurs at $\omega_{0m} \sim 200$ sec⁻¹, so $\gamma^2 = (\alpha/\omega_{0m})^2 = 0.01$, so γ^2 in (14) can be neglected ($\gamma^2 \ll 1$). If the resistance coefficient is taken as independent of the vibrational frequency, $\varkappa_m/\varkappa_\omega = 1$ and (14) becomes

$$\frac{D_\omega}{D_{\omega m}} = [\varkappa_m^2 (\eta^2 - 1)^2 + \eta^2]^{-1}. \tag{15}$$

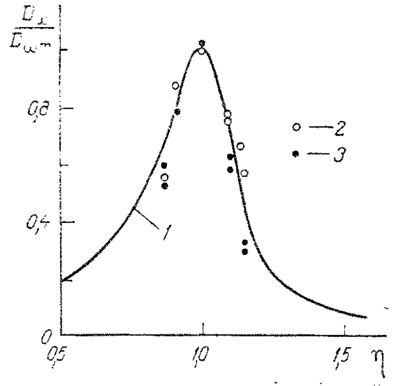

Fig. 2

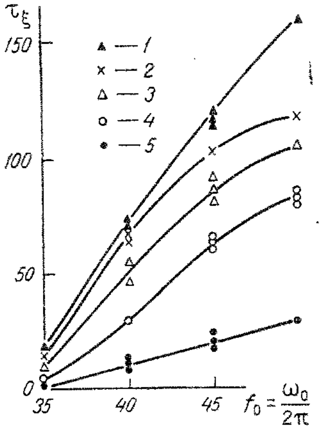

Fig. 3

Fig. 2. Dependence of $D_\omega/D_{\omega m}$ on η for a vibrated bed of electrocorundum: 1) from (15); 2) d = 0.07 mm; 3) 0.63 (A = 1.0-1.2 mm).

Fig. 3. Relaxation time τ_ξ (sec) as a function of vibration frequency $f_0 = \omega_0/(2\pi)$ (Hz) in a free vibrationally excited bed: 1) d = 1.25 mm; 2) 0.63; 3) 0.32; 4) 0.16; 5) 0.07 (A = 0.6-1.2 mm).

To test (14) and (15), one needs independent data on λ. Such data are lacking, so one employs semiempirical formulas such as the Ergan and Kozeni–Karman ones or other suitable relationships [13, 15-17].

Figure 2 shows for example that particles 0.07 mm in diameter give the best agreement with experiment for resistance coefficients given by [15]

$$2\lambda_\omega = \frac{18\nu\rho_f}{d^2\rho_p}\varphi(\varepsilon),$$

in which $\varphi(\varepsilon) = 1 + 3/2\,(1-\varepsilon)^{1/2} + 9/4\,(1-\varepsilon)^{2/3} + 15/8\,(1-\varepsilon)$, while for particles with diameter 0.63 mm, from [17] for Re < 10:

$$2\lambda_\omega = \frac{24}{\mathrm{Re}\,\varepsilon^{3.75}}. \tag{17}$$

As the resistance coefficients from (16) and (17) differ little, the (15) formulas for 0.07 and 0.63 mm diameters almost coincide, so a single curve is shown in Fig. 2, which also shows that the relative diffusion coefficients here $D_\omega/D_{\omega m}$ are almost independent of particle diameter. The measured points 2 and 3 in Fig. 2 for the calculated curve 1 well, so this concept on the thermal diffusivity in terms of D_ω defined by the small-scale random motion is justified.

Heat transfer between the particles and the environment is not incorporated in (15) because γ^2 is small. However, in the analytic description of the thermal diffusivity in such a bed, neglecting the internal heat transfer may mean that the measured a for the various x differ considerably, so one assumes that the random motion in an elementary volume with characteristic scale L >> d, and having temperature T entrains particles with a different temperatures, which gives internal heat transfer, which may be considered as relaxation to the equilibrium temperature $T^{(e)}$ [18].

We used the [19] method to describe the thermal conduction. We introduce ξ, which characterizes the disequilibrium: $\gamma\xi = (\Theta - \vartheta) = 1/T^{(e)} - 1/T$.

We make the usual assumption [20] that the entropy is dependent on the internal energy u and on ξ, $s = s[u(t), \xi(t)]$ with the standard quadratic formula

$$s = s^{(e)} - \frac{1}{2}h\xi^2, \quad h = -\left(\frac{\partial^2 s}{\partial \xi^2}\right)_{\xi=0}. \tag{18}$$

We differentiate (18) withrespect to time and use $(\partial s/\partial u)^{(e)} = 0$ to get the rate of change in entropy per unit volume:

$$\rho\dot{s} = \Theta\rho\dot{u} - \rho h\xi\dot{\xi}. \tag{19}$$

We determine $\rho\dot{u}$ and $\rho\dot{s}$ from the balance equations

$$\rho\dot{u} = -q_{x,x}, \quad \rho\dot{s} = -J^{(s)}_{x,x} + \sigma^{(s)} \tag{20}$$

(the comma denotes differentiation with respect to x) and substitute into (19) to get entropy production $\sigma^{(s)}$:

$$\sigma^{(s)} = q_x\vartheta_{,x} - (\Theta - \vartheta)\, q_{x,x} - \rho h\xi\dot{\xi}, \tag{21}$$

or from the definition of ξ

$$\sigma^{(s)} = q_x\vartheta_{,x} - \gamma\xi q_{x,x} - \rho h\xi\dot{\xi}. \tag{22}$$

Linear phenomenological laws and Fourier representations of them follow from (22):

$$q_x = L_{qq}\Theta_{,x}, \quad q_{\omega x} = L_{qq}\Theta_{\omega,x}, \tag{23}$$

$$\tau_\xi\dot{\xi} = -L_\xi\xi - \gamma q_{x,x}, \quad (1 + i\omega\tau_\xi)\,\xi_\omega = -\frac{\gamma}{L_\xi}\, q_{\omega x,x}, \tag{24}$$

in which the relaxation time τ_ξ is defined by $\rho h/L_\xi = \tau_\xi$, while the phenomenological coefficients for the thermal conductivity L_{qq} and relaxation L_ξ are taken as known.

To calculate h in (18), we use an expression for the entropy of a nonequilibrium system:

$$\rho s = \rho s^{(e)} + \rho c \ln|\Theta - \gamma\xi| + \frac{\gamma\rho c}{\Theta}\,\xi. \tag{25}$$

The first term in (25) corresponds to reversible transition between states having temperatures $T^{(e)}$ and T, and the second to the uncompensated heat [21]. At equilibrium ($\xi = 0$), the entropy is maximal, so $(\partial s/\partial\xi)_{\xi = 0} = 0$ implies

$$-\rho\left(\frac{\partial^2 s}{\partial\xi^2}\right)_{\xi=0} = \rho h = \frac{\rho c\gamma^2}{\Theta^2}, \tag{26}$$

with h = const during the relaxation.

Then (20) gives

$$\frac{\rho c}{\Theta^2}\,(\dot{\Theta} - \gamma\dot{\xi}) = q_{x,x}. \tag{27}$$

We transfer to Fourier components in (27) and use (26) to get

$$\frac{\rho h}{\gamma^2}\,(i\omega\Theta_\omega - i\omega\gamma\xi_\omega) = q_{\omega x,x}. \tag{28}$$

We use $q_{\omega x}$ from (23) in (28) and use (24) to eliminate ξ_ω to get up to terms in ω^2 that

$$\frac{\rho h}{\gamma^2}\,[i\omega\Theta_\omega + (i\omega)^2\,\tau_\xi\Theta_\omega] = L_{qq}\Theta_{\omega,x,x} \tag{29}$$

We transfer to the originals in (29) and take $\Theta = 1/T^{(e)} = 1/T$, to write the equation for the effective thermal conductivity:

$$\frac{\partial T}{\partial t} + \tau_\xi\frac{\partial^2 T}{\partial t^2} = a\frac{\partial^2 T}{\partial x^2}. \tag{30}$$

Here we have used $\tau_\xi = \rho h / L_\xi = \rho c \gamma^2 / (\Theta^2 L_\xi)$; $a = \lambda / (\rho c) = L_{qq} / (T^2 \rho c)$. (30) is used also to describe the thermal conduction in a fluidized bed [22]. In dimensionless form, (30) is

$$\frac{\partial \bar{T}}{\partial \tau} + \frac{\partial^2 \bar{T}}{\partial \tau^2} = \frac{\partial^2 \bar{T}}{\partial X^2}, \tag{31}$$

in which

$$\bar{T} = \frac{T(x,\ t) - T_0}{T^* - T_0}; \quad \tau = t/\tau_\xi; \quad X = x/\sqrt{a\tau_\xi}.$$

The boundary-value problem for the working conditions can be put as

$$\begin{gathered}
0 < x < R:\ T(0,\ x) = T^*,\ \ \bar{T}(0,\ X) = 1,\ \ \frac{\partial T(0,\ x)}{\partial t} = \frac{\partial \bar{T}}{\partial \tau} = 0; \\
R < x < \infty:\ T(0,\ x) = T_0,\ \ \bar{T}(0,\ X) = 0,\ \ \frac{\partial T(0,\ x)}{\partial t} = \frac{\partial \bar{T}}{\partial \tau} = 0; \\
t,\ \tau > 0:\ T(t,\ R-0) = T(t,\ R+0),\ \ \bar{T}(\tau,\ X_R - 0) = \bar{T}(\tau,\ X_R + 0), \\
X_R = R/\sqrt{a\tau_\xi}, \\
\frac{\partial T(t,\ 0)}{\partial x} = \frac{\partial \bar{T}(\tau,\ 0)}{\partial X} = 0; \quad \left.\frac{\partial T}{\partial x}\right|_{R-0} = \left.\frac{\partial T}{\partial x}\right|_{R+0}, \\
\left.\frac{\partial \bar{T}}{\partial X}\right|_{X_R - 0} = \left.\frac{\partial \bar{T}}{\partial X}\right|_{X_R + 0}.
\end{gathered} \tag{32}$$

Operational solution of (31) and (32) gives the transform $\bar{T} \risingdotseq \tilde{\bar{T}}$, which is

$$\tilde{\bar{T}} = \frac{1}{2p} \{\exp[-\sqrt{p(p+1)}\ (X - X_R)] - \exp[-\sqrt{p(p+1)}\ (X + X_R)]\}. \tag{33}$$

Inverting this transform gives [23]

$$\begin{aligned}
\bar{T}(x,\ t) = {} & \exp(-X/2)\,\mathrm{sh}\left(\frac{X_R}{2}\right) + \frac{X - X_R}{4} \int_{X - X_R}^{\tau} \exp(-y/2) \frac{I_1\left(\frac{z^{(-)}(y)}{2}\right)}{z^{(-)}(y)} dy - \\
& - \frac{X + X_R}{4} \int_{X + X_R}^{\tau} \exp(-y/2) \frac{I_1\left(\frac{z^{(+)}(y)}{2}\right) dy}{z^{(+)} y},
\end{aligned} \tag{34}$$

in which

$$z^{(-)}(y) = \sqrt{y^2 - (X - X_R)^2}; \quad z^{(+)} = \sqrt{y^2 - (X + X_R)^2}.$$

The maximum condition for (34) gives

$$\begin{aligned}
& \frac{(X - X_R)\, I_1\left(\frac{1}{2}\sqrt{\tau_m^2 - (X - X_R)^2}\right)}{\sqrt{\tau_m^2 - (X - X_R)^2}} = \\
& = \frac{(X + X_R)\, I_1\left(\frac{1}{2}\sqrt{\tau_m^2 - (X + X_R)^2}\right)}{\sqrt{\tau_m^2 - (X + X_R)^2}},
\end{aligned} \tag{35}$$

which can be used to determine the relaxation time, if one measures the time taken to attain the maximum temperature in a given section.

Figure 3 shows the relaxation time as a function of ω_0 from (35) for a free vibrationally excited bed, with a determined by experiment for the most remote section (the values of a for the two remote sections differed by an amount less than the error of experiment). Relaxation becomes more important as the particle diameter and vibrational frequency increase in the ranges used, so under certain conditions, the internal heat transfer should be described by (30) or (31).

The times t_m to attain the maximum temperature for sections at 170 and 210 mm in the free vibrationally excited bed were greater than τ_ξ, and the same applied to all the sections in the bed with tubes, so the method of determining the thermal diffusivity [3] from an ordinary parabolic equation is correct, although this does not mean that with rapid processes in a free vibrationally excited bed or in the presence of obstacles, it may not be necessary to use the hyperbolic equation.

NOTATION

T^*, initial bed temperature; T_0, environmental temperature; T_p, particle temperature; $T^{(e)}$, equilibrium temperature; $\bar{T}$, dimensionless temperature; θ , and ϑ, reciprocals of the current and dimensionless temperatures correspondingly; A vibration amplitude; ω_0, vibration frequency; Ω, gas oscillation frequency; H_0, bed height; d, particle diameter; g, acceleration due to gravity; a_0, and a, thermal diffusivities of immobile bed and effective value correspondingly; D_ω, analog of the turbulent thermal diffusivity coefficient; λ_{qf}, gas thermal conductivity; ν, gas kinematic viscosity; ρ_f, ρ_p, and ρ, densities of gas, particles, and bed correspondingly; ε, bed porosity; m_p, particle mass; P_1, P_2, and P_3, forces acting on a particle; f_p, perturbing force per kg of solid; c_p, specific heat of particle material; λ, resistance coefficient; u, internal energy; s, entropy; $J^{(s)}$, entropy flux; $\sigma^{(s)}$, entropy production; q, heat flux density; L_{qq}, and L_ξ, phenomenological coefficients for thermal conduction and relaxation correspondingly; ξ, relaxation parameter; τ_ξ, relaxation time, α, time-constant; x and X, dimensional and dimensionless coordinates correspondingly; t and τ, dimensional and dimensionless time correspondingly; $R_V(t)$, random-velocity autocorrelation function; $F_V(\omega)$, $F_X(\omega)$ spectral densities for random particle velocities and displacements correspondingly; $I_1(z)$, Bessel function; $\delta(\omega - \omega_0)$ Dirac δ function; $Re = \omega_0 d^2/\nu$ Reynolds number. A subscript m relates to the maximal value of the quantity.

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DIFFUSION APPROXIMATION FOR FLUIDIZED-BED COAL COMBUSTION

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UDC 662.61:66.096.5

A one-dimensional model has been constructed for stationary coal combustion, which is based on equations for the oxidant balance in the continuous and discrete phases together with the energy conservation equation for the burning particles in the fluidized bed. The model has been identified from measurements and parameters have been determined such as the mean particle size, the activation energy, and the gas and solid transport coefficients.

A fluidized-bed furnace opens up good economic prospects because it can burn a wide range of fuels efficiently while meeting tightening specifications on pollutant discharge. In such a furnace, the coal particles burn in an inhomogeneous fluidized bed of noncombustible material (the large fraction from the ash residue). On a two-phase model, an upward gas flow is necessary to start the fluidization, which breaks through as bubbles (discrete phase). The model concepts are fruitful and enable one to analyze commercial catalytic fluidized-bed reactors in which relatively slow heterogeneous reactions occur [1]. With a fast reaction, such as the combustion of solid fuel in a fluized bed, the model requires refinement. It has been suggested [2] that the gas bubbles are partially filled with burning particles and that the oxidation occurs in both phases.

A jet model is used for the combustion of coal having a high volatile content and entering the fluidized bed through the gas-distributing grid. The volatiles rapidly released from the fuel form combustible-gas jets at the grid, while the oxygen from the continuous phase diffuses to the jet boundaries [1].

A two-continuum model may be used to describe solid-fuel combustion in a fluidized bed composed of relatively coarse material, in which it is assumed that the entire gas flow is in contact with the solid and that no gas bubbles break through [3].

Here we present a three-continuum model. The bed consists of finely divided material (solid) suspended by the gas together with the gas, while in turn, the gas flow is divided into two continuous ones in accordance with the two-phase hydrodynamic fluidization theory: one is the gas passing through the channels between the grains (continuous phase) and the second is the gas in bubbles (discrete phase). A system of one-dimensional stationary nonlinear balance equations is formulated. The task if first simplified somewhat on the basis that the granular material mixes rapidly and therefore the burning-particle concentration and temperature remain virtually constant and are independent of the depth. One can therefore assume that the combustion rate does not vary with depth.

We write the conservation equation for the oxidant in the continuous and discrete phases:

$$k\varepsilon Y'' - \frac{1}{N-1} Y' - B\bar{\xi}\varphi YC + P\Pi (Y_{\text{d}} - Y) = 0, \tag{1}$$

$$Y'_{\text{d}} + P\Pi (Y_{\text{d}} - Y) = 0, \tag{2}$$

Lykov Heat and Mass Transfer Institute, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 4, pp. 618-623, April 1990. Original article submitted February 10, 1989.